AIAA JOURNAL
Vol. 35, No. 9, September 1997

Inverse and Direct Airfoil Design Using a Multiobjective Genetic Algorithm

Alessandro Vicini* and Domenico Quagliarella†
Centro Italiano Ricerche Aerospaziali, 81043 Capua, Italy

Some of the advantages and drawbacks of genetic algorithms applications to aerodynamic design are demonstrated. A numerical procedure for the aerodynamic design of transonic airfoils by means of genetic algorithms, with single-point, multipoint, and multiobjective optimization capabilities, is presented. In the first part, an investigation on the relative efficiency of different genetic operators combinations is carried out on an aerodynamic inverse design problem. It is shown how an appropriate tuning of the algorithm can provide improved performances, better adaption to design space size and topology, and variables cross correlation. In the second part, the multiobjective approach to design is introduced. The problem of the optimization of the drag rise characteristics of a transonic airfoil is addressed and dealt with using a single point, a multipoint, and a multiobjective approach. A comparison between the results obtained using the three different strategies is finally established, showing the advantages of multiobjective optimization.

I. Introduction

GENETIC search methods have their philosophical basis in Darwin's theory of survival of the fittest, inasmuch as they attempt to emulate the mechanisms that are typical of biological evolution.[1] Possible solutions to a given problem (individuals) are coded into bit strings, resembling the structure of chromosomes. A population of individuals, representing a set of design alternatives, is then allowed to reproduce and cross among the alternatives. The only information required by the procedure is a measure indicating the fitness of the individuals, i.e., how well each of them meets the design goal; this allows for a ranking of the individuals in the population. In the reproduction phase, the better individuals are promoted by using a selection criteria driven by the value of the fitness, so that bias can be allocated to the best fit members of the population. Genetic operators (crossover, mutation) are used to modify the chromosomes of the selected elements and generate the new offsprings, so that a combination of the most desirable characteristics of mating members is obtained; hence, subsequent generations produce elements characterized by higher fitness.[2]

A clear advantage in this approach is that the search does not require the computation of gradients. Moreover, genetic algorithms (GAs) accommodate all of the facets of soft computing, namely, robustness, nonlinearity, and uncertainty; the operations are domain independent; and a minimum effort is generally required in setting up the problem. These characteristics justify the emerging interest for GAs in engineering applications, including aerodynamic design problems.[3] The major drawback of evolutionary procedures lies at present in their computational efficiency; under this aspect, GAs cannot generally be compared to specialized approaches, such as automatic differentiation techniques or applications of control theory.[4,5] On the other hand, these methods require the development of flow solvers and optimization algorithms suited for the particular problem to deal with, offering very poor flexibility. General purpose optimization routines, based on the computation of gradients, can also provide good efficiency characteristics and have been widely used for engineering applications[6]; however, this approach generally requires human in the loop interaction and suffers from a number of drawbacks when highly multimodal functions or nonconvex optimization problems are dealt with.

Received Nov. 19, 1996; revision received May 19, 1997; accepted for publication May 20, 1997.

*Research Engineer, Transport Aircraft Aerodynamics Group, Via Maiorise. E-mail: a.vicini@cira.it.

†Research Engineer, Transport Aircraft Aerodynamics Group, Via Maiorise. E-mail: d.quagliarella@cira.it. Member AIAA.

In this paper the problem of airfoil design is addressed. Generally, all real-world design problems are ruled by several criteria that need to be satisfied, especially when multidisciplinary design environments are considered. Even considering the airfoil design problem from a purely aerodynamic point of view, requirements from different design points are to be taken into account to guarantee at least acceptable off-design performances. Hence, successful design methods must have multipoint design capabilities. Airfoil design methods may be divided into two general categories: inverse methods, where an inverse solver is used in at least part of the computational domain, and direct methods, where direct analyses are used in an iterative fashion. Inverse methods allow control on the detailed pressure distribution, but they are generally suited for single-point design only. Using a residual correction formulation,[7] multipoint design capability can be added to inverse methods by adding constraints on the appropriate parameters at off-design conditions, or by including the second design point in the objective function to be minimized. In Ref. 8, a target pressure distribution is used, which is obtained as a weighted average of the pressure distributions at the two design points. Alternative dual-point design procedures using an inverse design method have been used in Ref. 9; two different approaches are proposed, airfoil shape averaging and target pressure averaging, with the underlying assumption that aerodynamic characteristics vary linearly with changes in the airfoil shape. A possible hybrid approach is that of using parameters as design variables describing the pressure distribution and applying numerical optimization to find a pressure distribution that minimizes drag, subject to the desired constraints.[10] Direct approaches to design can be coupled with any analysis method and, thus, take advantage of the latest computational technology. With respect to inverse methods, they offer greater versatility for the enforcement of constraints, the possibility to include multiple requirements in the same objective function, and the ability to deal directly with the quantities of interest, i.e., aerodynamic coefficients, though user expertise may be necessary to take full advantage of this flexibility. In Ref. 11 drag is reduced at one design point while constrained at a second design point to be less than that of the base airfoil. In Refs. 12 and 13, optimization over a range of operating conditions is carried out by considering a weighted sum of the drag coefficients as an objective function at each flow condition. With this approach, several combinations of weights usually have to be tested in order to find a satisfactory compromise. Applications of GAs to airfoil design, including multipoint optimization, have been demonstrated in Refs. 14 and 15. In these works, the same described approach has been followed, by minimizing a scalar objective function where drag coefficients from different flow conditions are combined.

A peculiar feature of genetic algorithms, which will be discussed, is their capability to face multiobjective optimization; with this approach, a whole set of solutions of a multicriteria problem is developed, which meet the multiple design goals at different levels of compromise. In this way, the arbitrary choice regarding the weights to attribute to the different design criteria is avoided. Multiobjective optimization using GAs has already been used with success for a wide range of applications[16,17]; in Ref. 18, it has been applied to find the geometry of the airfoils that better approximate two conflicting target pressure distributions, specified for low-speed high-lift and transonic cruise conditions. In the present work, an application of multiobjective optimization for the improvement of the drag rise characteristics of a transonic airfoil will be shown and compared with the corresponding single and multipoint approaches.

II. Tuning of the Genetic Algorithm

Though GAs are robust and problem independent procedures, it may be useful to devote some attention to a better tuning of the algorithm to the specific problem at hand; in fact, different schemes for variables coding, different combinations of genetic operators, or the use of specific techniques can provide improved performances. Numerical investigation is often the best mean to accomplish a better tuning of a GA to the optimization problem. However, tuning via numerical experiments can be a tough task; in fact, due to the statistical nature of evolutionary procedures, it is necessary to gather data from several runs, characterized by different initial conditions, i.e., different starting generations, to draw conclusions regarding the behavior of a particular procedure. This can be accomplished, of course, if the problem is not too computationally expensive; in the case of aerodynamic optimization, generally, it is not practical to undertake extensive investigations. For this reason, the use of test problems characterized by cheap cost functions for the analysis of the performances of different GAs is widespread[19]; on the other hand, if the test problem chosen is not representative of the one to be faced, in terms of fitness landscape, cross correlation, and physical meaning of the design variables, etc., the results that are found may be misleading.[20]

Some results will be illustrated regarding the inverse design of a transonic airfoil, with the purpose of assessing the merits and shortcomings of GAs for a highly nonlinear problem typical for the aerodynamic design field of application. Eight different strategies have been used for the GA, characterized by a different combination of mutation, crossover, and selection operators, as shown in Table 1.

Mutation was alternatively performed at bit or word level, i.e., directly on the bits of the binary coding, or on the integer number ranging from 0 to $2^n - 1$, n being the number of bits adopted for each variable, according to the following scheme:

$$\underbrace{x_i}_{\text{continuous variable}} \rightarrow \underbrace{k_i \in [0, 2^n - 1]}_{\text{word level coding}} \rightarrow \underbrace{[0110 \ldots 1011]}_{\text{bit level coding}} \tag{1}$$

It can be observed how, beyond the different behavior of these two operators, mutation probability assumes a different meaning in the two cases; in fact, for an assigned mutation probability p, the total probability of a n-bit string (representing one design variable) to undergo mutation is $P = 1 - (1 - p)^n$, whereas acting on the word we have $P = p$. As an alternative to the standard one point crossover, acting on the bit coded strings, the extended intermediate crossover (EIR), described in Ref. 21, was used. Let $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ be the selected parent chromosomes; with extended intermediate recombination, the offspring variable is given by

$$z_i = x_i + \alpha_i(y_i - x_i) \qquad i = 1, \ldots, n \tag{2}$$

where α_i is randomly chosen in the interval $[-d, 1 + d]$; $d = 0.2$ was used for the present study, and the preceding relation was used at word level coding of the variables. Finally, two different selection methods have been adopted, roulette wheel and random walk. With the first one, the selection probability assigned to each individual is proportional to its fitness. In the second one, the population is distributed over a toroidal landscape (Fig. 1), a starting point in the landscape is chosen at random, and the first parent selected is the best fit individual met in a random walk of assigned number of steps from that starting point; the second parent is selected in a successive random walk from the same starting point. A four-step random walk was used in the present analysis. In each run, the elitist strategy was used by explicitly transferring the best fit individual of each generation in the successive one.

Table 1 Strategies of genetic algorithm

GA no.	Mutation	Selection	Crossover
1	Bit	Random walk	EIR
2	Bit	Random walk	One point
3	Bit	Roulette wheel	EIR
4	Bit	Roulette wheel	One point
5	Word	Random walk	EIR
6	Word	Random walk	One point
7	Word	Roulette wheel	EIR
8	Word	Roulette wheel	One point

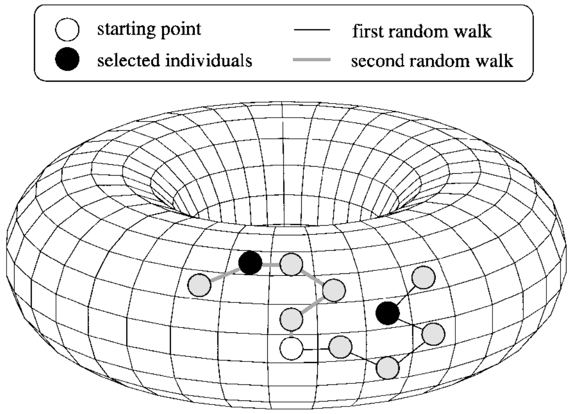

Fig. 1 Sketch of the mechanism used for random walk selection; each square on the torus contains an individual.

In the inverse design problem, a pressure distribution is given corresponding to a design point (Mach, α), and the geometry of the airfoil producing this target pressure distribution must be found. In this case, the objective function to be minimized is computed by

$$\text{obj} = 10 \int_S \left(c_p - c_p^{(t)}\right)^2 \mathrm{d}S \tag{3}$$

where c_p and $c_p^{(t)}$ are the actual and target pressure distributions, respectively, and S is the actual airfoil contour. A full potential transonic flow solver, with nonconservative formulation, has been used to calculate the flowfield. The fitness is obtained as $f = 1/\text{obj}^2$, which also produces a fitness scaling effect: the same difference in the objective function originates bigger differences in the corresponding fitnesses, when the objective approaches zero; this affects selection through the roulette wheel selection method, but not through the random walk method. The airfoil geometry is represented by means of two fifth-order B-spline curves, for the upper and lower parts; the coordinates of the control points of the B-spline constitute the design variables.[22] Initially, seven control points are used both for the upper and lower surfaces of the airfoil, including those fixed at the leading and trailing edges, for a total of 18 design variables (the first control points at the leading edge can move only in direction y); in the second phase of the run, the accuracy of the geometry representation is increased by using eight control points, for a total of 22 design variables. The use of less control points at the beginning of the design process reduces the complexity of the problem (which is strongly nonlinear), thus increasing the convergence rate; a suboptimal solution is more rapidly found and is then refined in the second part of the run. The actual problem here presented consists in the reconstruction of the CAST-10 airfoil[23] at $M = 0.765$ and $\alpha = 0$. This problem has been solved using a NACA 0012 as the initial guess, which can be considered an absolutely generic starting point.

Figure 2 shows the geometry of the target airfoil, the B-spline representing the NACA 0012 at the beginning of the run, and the

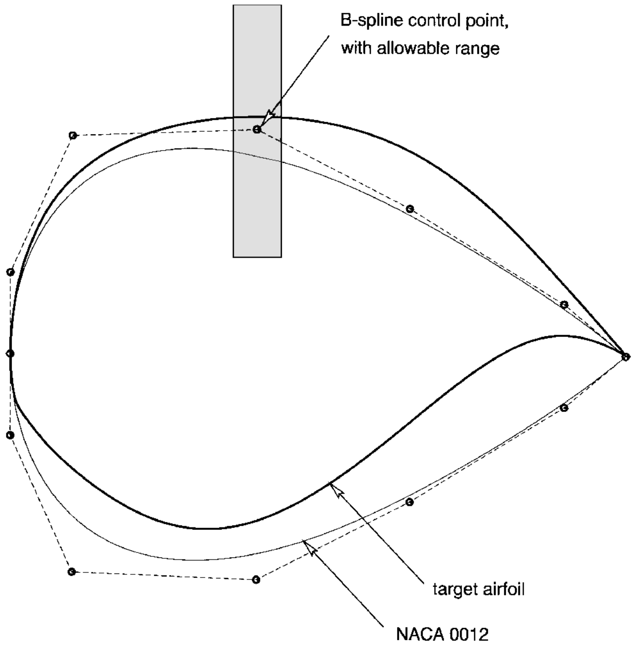

Fig. 2 Geometry of the target airfoil and B-spline used to represent the NACA 0012, with allowable range for control points.

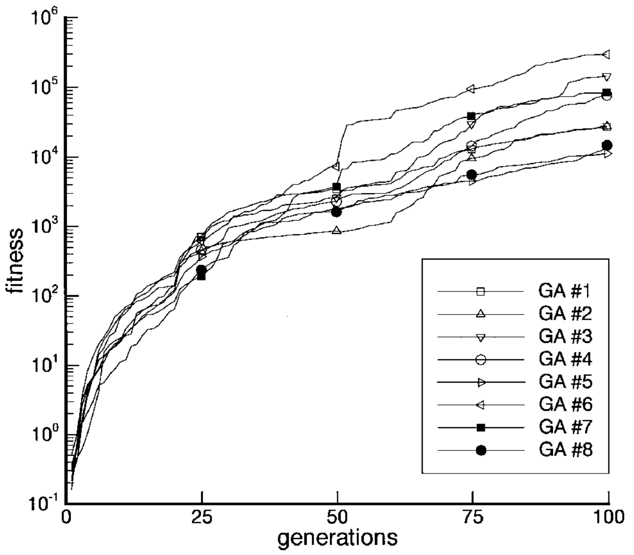

Fig. 3 CAST-10 airfoil reconstruction; evolution histories averaged over 10 trials, starting from NACA 0012.

allowable range of variation of the control points; as can be seen, the range is very wide, so that no specific knowledge of the final desired shape is used. Figure 3 illustrates the convergence histories, each one averaged over 10 different trials, obtained with the strategies of Table 1. For each case, crossover and mutation probabilities were set to 1 and 0.02 respectively, 8 bits were used for variables coding, and a 50 individuals population evolved for 100 generations. The control points refinement was applied after the 20th generation, and a restriction of the allowable ranges for the design variables was applied at generation 60. No convergence criteria were used; the comparison is carried out for the same number of generations. For a case like this, a fitness value of 10^4 can be considered acceptable as a converged result; from this point of view, all of the results obtained are converged. It can be seen that using the best performing strategies makes it possible to obtain a converged solution in about half the generations, with respect to the other ones. Of course, a statistical analysis based on more trials would produce more reliable results, but nevertheless it seems the 10 runs considered here are enough to show the significant tendencies. Thus, it can be concluded that, for this problem, when mutation is applied at bit level, the higher selection pressure obtained with roulette wheel selection is beneficial; in this case, the EIR crossover performs better than the one point. When using mutation at word level instead of bit level, the preservation of population diversity obtained through the random walk operator has a better effect, in conjunction with the one point crossover. In Fig. 4 one of the pressure distributions obtained, corresponding to a fitness of the order of 10^5, is shown together with the target and initial ones.

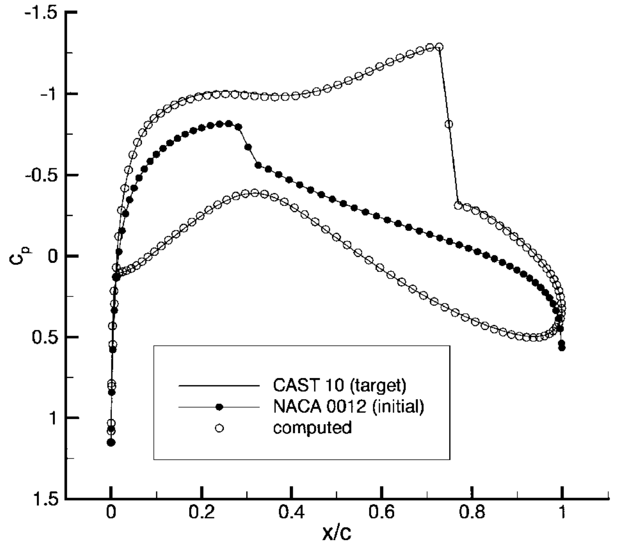

Fig. 4 Pressure distributions at $M = 0.765$, $\alpha = 0$ deg over the CAST-10 and NACA 0012 airfoils, with one of the results obtained.

As a concluding remark, we note that the described application emphasizes one weakness of GAs. In fact, GAs are very good at effectively exploring very large design spaces and rapidly locating a suboptimal solution but are not suited for the subsequent refinement of this solution; this can be done, but at the expense of a high computational cost, as seen with the results illustrated. Though the NACA 0012 is very far from the desired target, 3–6 generations are sufficient to obtain fitness values of the order of 10; the corresponding airfoil is already close to the target one. Then, convergence rate slows down in the subsequent generations. The calculations illustrated have been performed on a single processor of an SGI Power Challenge GR R-1000, where each of the described runs takes about 2 h of CPU time. A classical optimization method based on conjugate gradients can solve a problem such as that illustrated with a number of flow solutions of the order of $\frac{1}{5}$ of those required by the best GA,[14] provided that a suitable starting point is assigned.

III. Multiobjective Optimization

When an optimization problem is governed by several criteria, it is generally possible to combine these into a single number, usually through a weighted linear combination of the different objectives, or on the basis of a demand-level vector[24]; the problem becomes in this way amenable to all classical optimization algorithms. The drawback of this approach (referred to as multipoint) is that the solution of the problem is dependent on the (arbitrary) choice of the relative weights assigned to the objectives; moreover, if the objectives to be optimized are of different nature, as happens, for example, when multidisciplinary optimization problems are faced, it is difficult to understand how to interrelate them properly.

It can be convenient to follow a different approach by classifying all potential solutions to the problem into dominated and nondominated (Pareto optimal) solutions. To define the notion of domination, let $\boldsymbol{F} = (f_1, \dots, f_n)$, the vector of a minimization problem with n objectives, and let F^a and F^b be two candidates; F^a dominates F^b if

$$\forall i \in \{1, \dots, n\} \qquad f_i^a \le f_i^b \tag{4}$$

and

$$\exists i \mid f_i^a < f_i^b \tag{5}$$

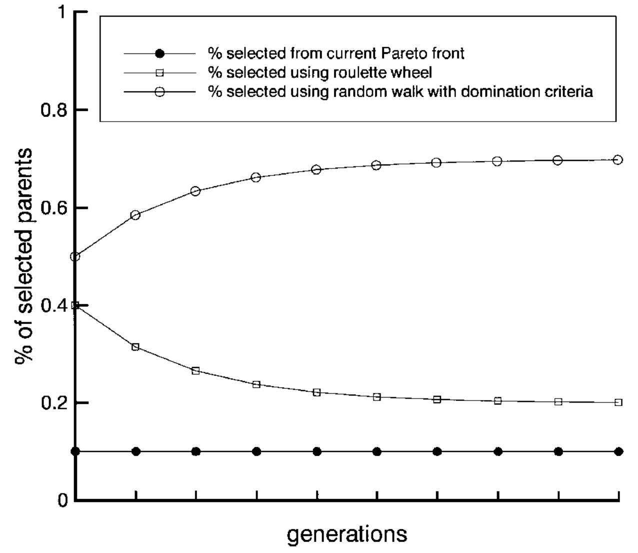

Fig. 5 Parents selection strategy for multiobjective optimization; at each generation, the sum of the three curves equals 1.

The Pareto front is the set of all of the nondominated solutions; it follows from the definition that if a solution belongs to the Pareto front it is not possible to improve one of the objectives without deteriorating some of the others.

By virtue of their structure, GAs are capable of facing multiobjective design problems in a more straightforward way. The characterizing feature of a multiobjective GA is the introduction of the Pareto criteria in the method used for individuals selection; by selecting individuals in the reproduction phase according to the domination criteria, a whole set of Pareto optimal solutions can be developed. These are all possible alternative solutions to the problem, which meet the requirements at different levels of compromise.[25]

In this work, multiobjective optimization is accomplished by using the same kind of random walk operator described in Sec. II; in this case, the elements selected are not the best fit ones but the nondominated among those met in the walk. If more nondominated solutions are met, the first one encountered is selected. At the end of every new generation, the set of Pareto optimal solutions is updated and stored.

A sort of extension of the elitism strategy to multiobjective optimization can also be adopted by randomly selecting an assigned percentage of parents from the actual set of nondominated solutions. Moreover, the possibility of selecting a specified percentage of individuals through a roulette-wheel operator, acting on one of the objectives of the optimization problem randomly chosen, has been introduced. This increases selection pressure toward the minima of the objectives of the problem considered individually, improving the convergence rate in the first phase of the front development. The amount of parents selected by roulette wheel selection can possibly be decreased as evolution proceeds, as shown in Fig. 5.

IV. Airfoil Drag Rise Optimization

The optimization problem here presented is the improvement of the drag rise characteristics of a transonic airfoil. As already stated, a single-point design generally leads to unacceptable off-design performances; therefore, it is necessary to specify multiple (at least two) design points; one possible approach, which has been followed in this work, is that of specifying the operating conditions at each design point in terms of Mach number and lift coefficient and trying to reduce the corresponding drag coefficient. Results will be presented pertinent to three different approaches: the first one is a single-point design; then, a two-point optimization is illustrated, carried out by means of both a multipoint and a multiobjective strategies.

The airfoil chosen as starting point for the optimization is the RAE 2822 (Ref. 26), and a design lift coefficient $c_l = 0.88$ has been fixed. The constraint on lift coefficient is satisfied by including the angle of attack among the design variables and letting the flow solver seek for the angle of attack producing the desired lift. This approach has been preferred to the explicit use of a constraint, which could be enforced, for example, by adding a penalty function to the value of the fitness. The main reason for this choice is that, in the transonic regime, a very small change in lift coefficient may cause abrupt changes in the corresponding drag; hence, for a more significant comparison, it is important that all solutions taken into consideration are characterized by the same lift coefficient. For this reason, an equality constraint should be used; however, this cannot be realized with a penalty function, and it is generally also difficult to implement with classical numerical optimization techniques. From the point of view of computational efficiency, the approach followed requires a bigger effort for the analysis of each airfoil; on the other hand, none of the analyzed airfoils must then be discarded for not satisfying the constraint; therefore, a smaller total number of airfoils needs to be considered.

To avoid trivial solutions, another constraint has been imposed on the airfoil thickness, fixed at 12%, by automatically scaling the airfoil to the desired thickness value after each geometry modification, before being analyzed. With respect to the case of inverse design, a different approach has been followed for geometry manipulation by using shape functions[22,27]; the airfoil geometry is obtained as a linear combination of the baseline shape and a number of given modification functions, the coefficients of the combination being the design variables. The use of shape functions, according to the authors' experience, usually proves more effective when drag minimization is the objective, as they are more suited for smaller and more specialized modifications, which are generally needed in these cases. For the examples described, 15 design variables have been used for the upper side of the airfoil and 16 for the lower one.

A. Single-Point Design

Initially, three different single-point designs have been carried out, at $M = 0.72$, 0.73, and 0.74, respectively, and at the desired lift $c_l = 0.88$. The objective function is computed as $\text{obj} = 100c_d/c_l^2$ (where wave drag is the only contribution) and, as in the inverse design case earlier described, the fitness is then obtained as $f = 1/\text{obj}^2$. The wave drag is calculated by integrating the momentum loss across the shock waves. Crossover probability has been set to 1 with the one point operator, and mutation probability has been set to 0.01 at word level; selection has been carried out by roulette wheel, and a population size of 50 individuals has been used. The single-point design is a relatively easy task, and an optimum solution characterized by shockless flow at the design point generally can be found (besides, it must be noted that the solution to this problem is not unique). The shockless solution was found after 7 generations at $M = 0.72$, 6 generations at $M = 0.73$, and 50 generations at $M = 0.74$. As anticipated, these solutions are characterized by a deterioration of the performances at off-design conditions; this is confirmed by Fig. 6, showing the comparison of the drag rise curve of the original RAE 2822 airfoil, at the design lift coefficient, with those of the three

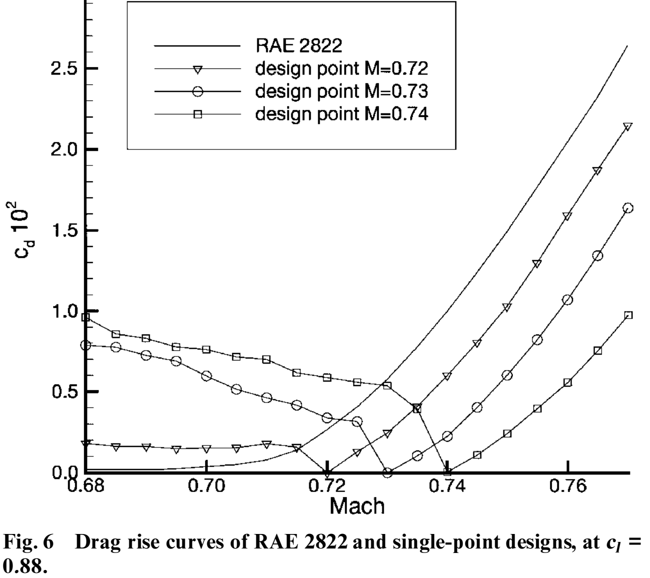

Fig. 6 Drag rise curves of RAE 2822 and single-point designs, at c_l = 0.88.

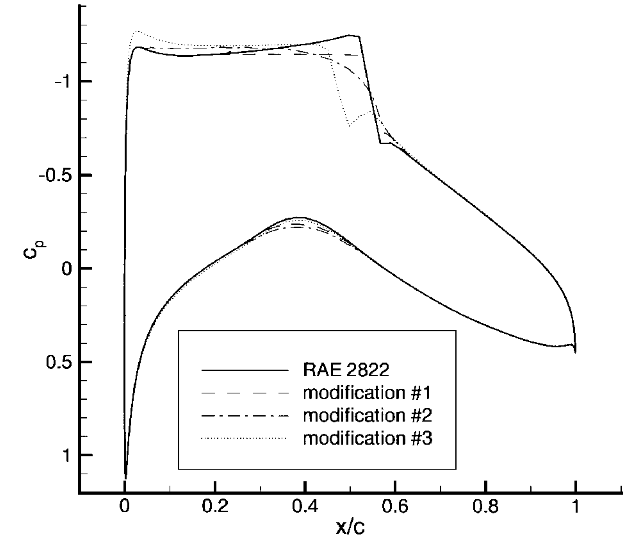

Fig. 7 Pressure distribution over the RAE 2822 at $M = 0.72$, $c_l = 0.88$, and three proposed modifications.

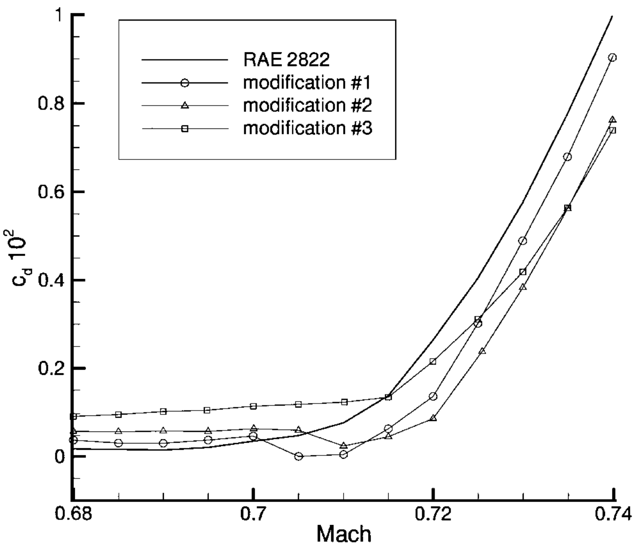

Fig. 8 Drag rise curves of RAE 2822 and single-point inverse designs, at $c_l = 0.88$.

solutions found. It is apparent how these are characterized by a significant drag creep and must, therefore, be discarded.

A possible alternative single-point approach is that of recurring to inverse design. This can be done as follows: a design Mach number is chosen, and the pressure distribution over the original airfoil is computed at the design lift coefficient. This pressure distribution is then manually modified so as to reduce its drag, and the corresponding airfoil is obtained through inverse design. In this case a design Mach number $M = 0.72$ has been chosen; in Fig. 7 the corresponding pressure distribution over the RAE 2822 is shown with three possible modifications. In the first one, only a slight reduction of shock wave intensity has been introduced; in the second one, an almost shockless recompression has been prescribed; in the third one, the pressure distribution produced by the RAE 2822 at $M = 0.71$ is assigned. In these cases, of course, there is no guarantee that an exact solution to the inverse problem exists; in general, an approximation of the specified target is obtained. Using the B-spline representation and the same parameters of strategy 6 in Table 1, 5–10 generations of 50 individuals are enough to find the airfoils corresponding to the three modifications described (having also introduced the constraint on maximum thickness). The drag rise curves of these airfoils are shown in Fig. 8. As can be observed, these solutions also have "single point" characteristics, producing an increase of drag at lower Mach numbers that is higher for the solutions corresponding to bigger modifications of the original c_p distribution.

B. Multipoint Design

The second step consisted in a two-point design, with the objective function computed as a weighted sum of c_d/c_l^2 at the two design points: $\text{obj} = 100[\alpha(c_d/c_l^2)_1 + (1 - \alpha)(c_d/c_l^2)_2]$; $M = 0.7$ and 0.75 were chosen as design points, at the same lift coefficient $c_l = 0.88$ earlier considered. Three different runs have been carried out, with $\alpha = 0.33$, 0.50, and 0.66, respectively. The same GA parameters as in the single-point case have been used, except for individual selection, which has been carried out with a four step random walk.

Figure 9 shows the convergence histories, in terms of maximum fitness, obtained with a population of 50 individuals; convergence is relatively fast except for the case $\alpha = 0.50$, where 100 generations are needed. In Fig. 10, the drag rise curves of the three final airfoils are shown (at the design lift coefficient). Two of the solutions obtained are characterized by an increase of drag at Mach numbers lower than the design ones; this is apparent in the case $\alpha = 0.50$, but also with $\alpha = 0.66$ we have 5 drag counts at $M = 0.68$. The solution obtained with $\alpha = 0.33$ can be considered the best one; it is characterized by a wave drag that is very close to that of the original airfoil up to $M = 0.71$ (within 1 drag count) and becomes lower at higher Mach numbers. At $M = 0.72$ we have a reduction of wave drag of 16 counts. At the second design point, $M = 0.75$, the solution is shockless. It is apparent, as already pointed out, how the solution to the problem formulated in this way is largely dependent on the

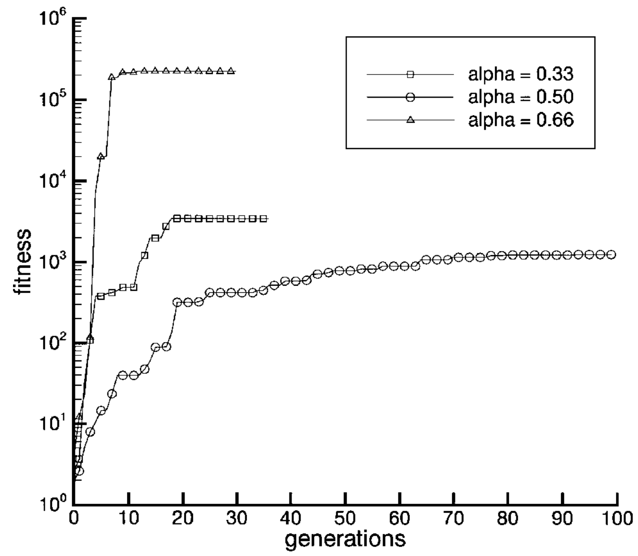

Fig. 9 Convergence histories of multipoint designs in terms of maximum fitness.

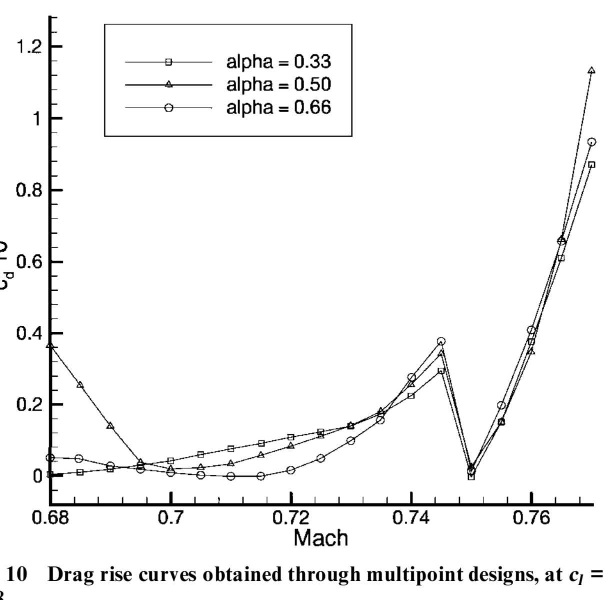

Fig. 10 Drag rise curves obtained through multipoint designs, at $c_l = 0.88$.

values of the weights used in the objective function, without the possibility of understanding in advance which weights will produce a solution of the desired characteristics. A whole set of different values should be tested and the corresponding solutions analyzed in a trial and error process until a satisfactory one is found.

C. Multiobjective Design

In the multiobjective design process, the same parameters as in the preceding run have been used, except for individuals selection, which has been carried out following the mechanism already described and shown in Fig. 5. The probability of selection from the current Pareto front has been set to 0.1, whereas that of selection through roulette wheel varied from a value of 0.4 at the beginning of the process to 0.2 at the end. A population of 100 individuals evolved for 80 generations. The whole process required about 15 h of CPU time on a SGI Power Challenge GR R-1000. Figure 11 shows the Pareto front of the nondominated individuals after generations 1, 2, and 4, and at the end of the run (in terms of the objective functions to be minimized). It can be seen how the front evolves in a relatively rapid fashion at the beginning of the run, because of the use of roulette wheel in the selection mechanism; the two solutions corresponding to $obj_1 = 0$ and $obj_2 = 0$, respectively, are found after the fourth generation. The second phase of the process is devoted to the development of a more numerous and uniform distribution of nondominated individuals. The final Pareto front is populated by 20 individuals; it can be seen from Fig. 11 how the solutions at its extremities, corresponding to $obj_1 = 0$ and $obj_2 = 0$, respectively, have been improved with regard to the other objective, with respect to those initially found. The characteristics of the shockless solution at $M = 0.75$ are similar to what was found with the single-point designs, i.e., an increase of wave drag is obtained at Mach numbers lower than 0.7 with respect to the original airfoil. On the other hand, the solution at the opposite extremity of the front, shockless at $M = 0.7$, can actually be considered an improvement over the RAE 2822; an analysis of the other elements lying on the front reveals that other solutions exist that can be considered as a better compromise. In particular, Fig. 12 shows the drag rise curve of the airfoil that can be considered the best among those found, compared to that obtained with the multipoint design and to the original airfoil. In Fig. 13 the difference between the wave drag coefficient of the RAE 2822 and that of the best solutions obtained with the multipoint and multiobjective designs is shown. It can be observed how the multiobjective design is superior to the multipoint up to $M = 0.74$; at this Mach number, the drag reduction with respect to the RAE 2822 is equal to 78 drag counts. The characteristic hole shown by the drag polars of all solutions at $M = 0.75$ could possibly be avoided by a different choice of the design points, or by adding a third design point at an intermediate Mach number. In Fig. 14 the pressure distribution at $M = 0.72$ and $c_l = 0.88$ is shown for the RAE 2822 and the optimum multipoint and multiobjective designs.

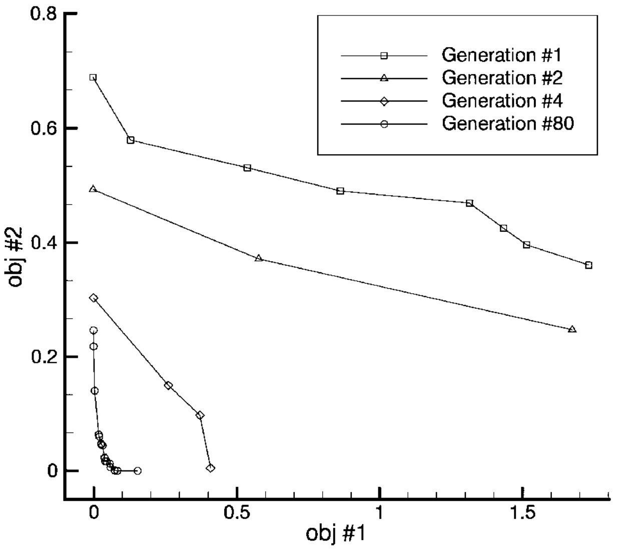

Fig. 11 Pareto fronts of multiobjective drag rise optimization.

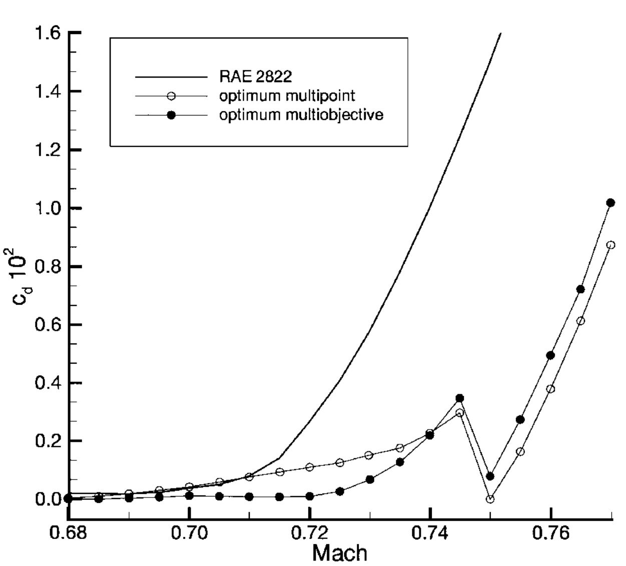

Fig. 12 Comparison between drag rise curves of RAE 2822, multipoint, and multiobjective designs, c_l = 0.88.

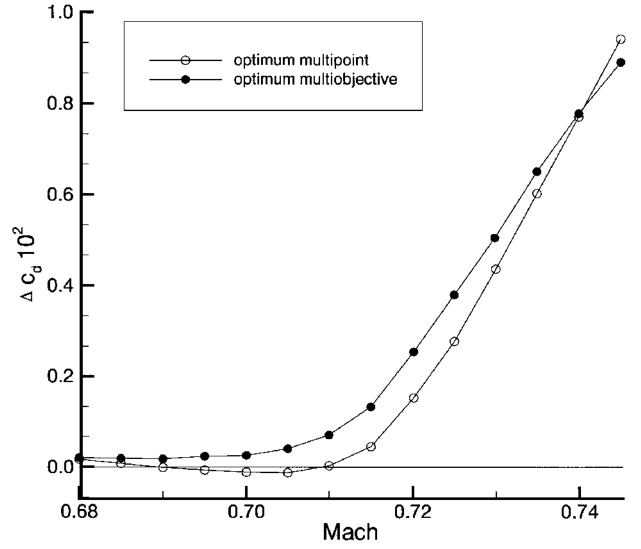

Fig. 13 Drag reduction obtained through multipoint and multiobjective designs with respect to RAE 2822.

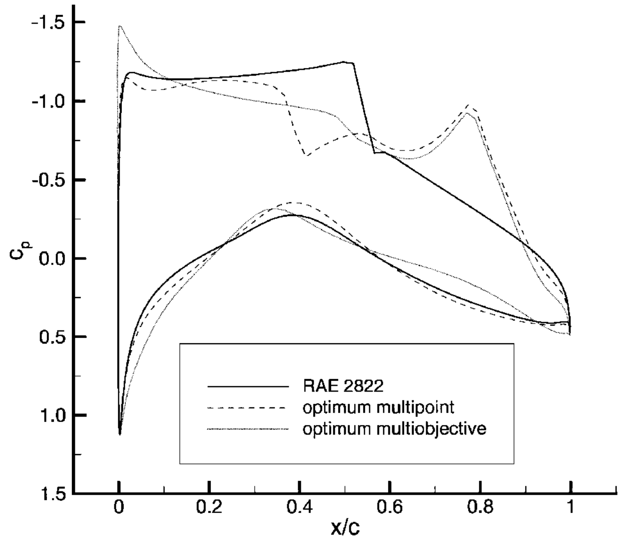

Fig. 14 Pressure coefficient distribution at M = 0.72, c_l = 0.88 for the RAE 2822, optimum multipoint, and multiobjective designs.

One possible weakness of the followed approach clearly is the lack of control on the quality of the pressure distribution; moreover, for an optimization of practical interest, viscous effects should be taken into account, and other constraints should possibly be considered, to keep under control, for example, pitching moment, leading-edge radius, and trailing-edge thickness.

V. Conclusions

A procedure based on GAs and devoted to the inverse or direct design of airfoils has been presented. The main advantage of the genetic approach is that the gradients are not required in the search process; hence, the method is not susceptible to the pitfalls of hill-climbing techniques. From the applications that have been illustrated, it appears that GAs do not constitute by themselves an alternative to more traditional optimization methods; rather, they can be considered as valid complementary techniques, being characterized by peculiar features that can be conveniently exploited. The major drawback lies in their computational efficiency. In particular, as seen in the case of inverse airfoil design, the computational cost can be extremely high if a fully converged optimum design needs to be obtained. From this point of view, it has been shown how a significant efficiency improvement can be obtained by a proper selection of genetic operators; moreover, thanks to their structure, GAs can take full advantage of parallel computing. On the other hand, the same application mentioned shows one of the favorable feature of GAs, namely, their capability of exploring the design space, rapidly identifying the region of the global optimum. This is a very important characteristic, especially when dealing with nonconvex or disjoint design spaces; besides, unlike the gradient-based approach, increasing the dimensionality by adding design variables does not impose a proportional computational cost.

Of particular importance is the case when several criteria are to be satisfied. The capability of facing multiobjective optimization in a straightforward fashion, without the need of arbitrarily interrelating the multiple criteria, appears of great interest. The designer can choose among a set of alternative (nondominated) solutions; this allows better control of off-design performances and adds flexibility to the design procedure; for example, the multiobjective approach can be conveniently adopted to manage design constraints. The advantages with respect to single objective optimization have been shown in the application example. The most favorable capabilities of GAs might be favorably exploited through a proper hybrid scheme, combining them with an optimization technique of different nature, so that the best feature of both methods can be utilized. Hybrid schemes have already been shown to be promising for multiobjective optimization[18] and can open the way for the use of GAs for more complex problems, such as three-dimensional aerodynamic optimization.

References

[1]Holland, J. H., *Adaptation in Natural and Artificial Systems*, Univ. of Michigan Press, Ann Arbor, MI, 1975, Chap. 1.

[2]Goldberg, D. E., *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison–Wesley, Reading, MA, 1989, pp. 175–180.

[3]Ahmed, Q., Krishnakumar, K., and Neidhoefer, J., "Applications of Evolutionary Algorithms to Aerospace Problems—A Survey," *Computational Methods in Applied Sciences '96*, Wiley, London, 1996, pp. 236–242.

[4]Topliss, M. E., Toomer, C. A., and Hills, D. P., "Rapid Design Space Approximation for Two-Dimensional Transonic Aerofoil Design," *Journal of Aircraft*, Vol. 33, No. 6, 1996, pp. 1101–1108.

[5]Jameson, A., "Aerodynamic Design via Control Theory," *Journal of Scientific Computing*, Vol. 3, Sept. 1988, pp. 233–260.

[6]Cosentino, G. B., and Holst, T. L., "Numerical Optimization Design of Advanced Transonic Wing Configurations," *Journal of Aircraft*, Vol. 23, No. 3, 1986, pp. 192–199.

[7]Volpe, G., "Transonic Shock Free Wing Design," AGARD R-780, Paper 5, Nov. 1990.

[8]Labrujère, T. E., "Residual-Correction Type and Related Computational Methods for Aerodynamic Design," *Optimum Design Methods in Aerodynamics*, Von Kármán Inst. Special Course, Rhode-Saint-Genèse, Belgium, 1994.

[9]Mineck, R. E., Campbell, R. L., and Allison, D. O., "Application of Two Procedures for Dual-Point Design of Transonic Airfoils," NASA TP-3466, Sept. 1994.

[10]Van den Dam, R. F., van Egmond, J. A., and Sloof, J. W., "Optimization of Target Pressure Distributions," AGARD R-780, Paper 3, Nov. 1990.

[11]Hinson, M. L., "A Series of Airfoils Designed by Transonic Drag Minimization for Gates Learjet Aircraft," *Transonic Aerodynamics*, edited by D. Nixon, AIAA, New York, 1982, pp. 489–509.

[12]Hicks, R. M., and Vanderplaats, G. N., "Application of Numerical Optimization to the Design of Supercritical Airfoils Without Drag-Creep," Society of Automotive Engineers, SAE Paper 770440, March 1977.

[13]Hager, J. O., Eyi, S., and Lee, K. D., "Two-Point Transonic Airfoil Design Using Optimization for Improved Off-Design Performances," *Journal of Aircraft*, Vol. 31, No. 5, 1994, pp. 1143–1147.

[14]Quagliarella, D., "Genetic Algorithms Applications in Computational Fluid Dynamics," *Genetic Algorithms in Engineering and Computer Science*, edited by G. Winter, J. Périaux, M. Galán, and P. Cuesta, Wiley, London, 1995, pp. 417–442.

[15]Quagliarella, D., and Della Cioppa, A., "Genetic Algorithms Applied to the Aerodynamic Design of Transonic Airfoils," *Journal of Aircraft*, Vol. 32, No. 4, 1995, pp. 889, 890.

[16]Rao, S. S., "Genetic Algorithmic Approach for Multi Objective Optimization of Structures," *Structures and Controls Optimization*, edited by N. S. Khot and R. T. Haftka, AE Series Vol. 38, American Society of Mechanical Engineers, New York, 1993, pp. 29–38.

[17]Krishankumar, K., Swaminathan, R., and Montgomery, L., "Multiple Optimal Solutions for Structural Control Using Genetic Algorithms with Niching," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 6, 1994, pp. 1374–1377.

[18]Quagliarella, D., and Vicini, A., "Hybrid Genetic Algorithms as Tools for Complex Optimization Problems," Centro Italiano Ricerche Aerospaziali, CIRA TR-97057, Capua, Italy, May 1997.

[19]Yamamoto, K., and Inoue, O., "New Evolutionary Direction Operator for Genetic Algorithms," *AIAA Journal*, Vol. 33, No. 10, 1995, pp. 1990–1993.

[20]Vicini, A., and Quagliarella, D., "Multipoint Transonic Airfoil Design by Means of a Multiobjective Genetic Algorithm," AIAA Paper 97-0082, Jan. 1997.

[21]Mühlenbein, H., "The Science of Breeding and Its Application to Genetic Algorithms," *Genetic Algorithms in Engineering and Computer Science*, edited by G. Winter, J. Périaux, M. Galán, and P. Cuesta, Wiley, London, 1995, pp. 59–82.

[22]Quagliarella, D., and Vicini, A., "Transonic Airfoil Design by Means of a Genetic Algorithm," *Proceedings of 13th AIDAA Congress*, Associazione Italiana di Aeronautica ed Astronautica, Rome, 1995, pp. 933–942.

[23]Ray, E. J., and Hill, A. S., "CAST-10-2/DOA 2, Airfoil Studies Workshop Results," NASA CP 3052, Nov. 1989.

[24]Michalewicz, Z., *Genetic Algorithms + Data Structures = Evolution Programs*, Springer–Verlag, Berlin, 1994, pp. 175–180.

[25]Poloni, C., Mosetti, G., and Contessi, S., "Multi Objective Optimisation by GAs: Application to System and Component Design," *Computational Methods in Applied Sciences '96*, Wiley, London, 1996, pp. 258–264.

[26]Cook, P. H., McDonald, M. A., and Firmin, M. C. P., "Aerofoil RAE 2822—Pressure Distributions, Boundary Layer and Wake Measurements," AGARD AR-138, Paper A6, May 1979.

[27]Hicks, R., and Henne, P. A., "Wing Design by Numerical Optimization," *Journal of Aircraft*, Vol. 15, No. 7, 1978, pp. 407–412.

A. Plotkin
Associate Editor